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ENTRAINMENT FOR HORIZONTAL ANNULAR GAS–LIQUID FLOW

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Abstract—Measurements of entrainment are presented for air and water flowing in horizontal 2.54 and 5.08 cm pipelines. After the initiation of atomization, entrainment increases with the third power of the gas velocity. At very high gas velocities a fully entrained condition is reached for which further increases in the gas velocity do not cause a decrease in the flow rate of the wall film. Gas density has a small effect provided comparisons are made at the same gas velocity rather than at the same mass flowrate. The results are interpreted by assuming that the rate of deposition of droplets on the wall film varies linearly with the concentration of droplets and that the rate of atomization of the wall film varies linearly with its flow rate.

1. INTRODUCTION

It is now recognized that major improvements in the design methods for gas–liquid flow in a pipeline, suggested by Martinelli over thirty years ago, must take account of how the gas and liquid are distributed. Progress in this direction has been slow, mainly because quantitative information on the configuration of the phases over a pipe cross-section, and the physical processes responsible for this configuration, is not adequate.

For this reason, we have undertaken a detailed study of the annular regime that exists for gas–liquid flows in horizontal pipelines at large gas velocities. The system being used consists of two pipelines, 2.54 and 5.08 cm in diameter, with lengths up to 2740 cm. The fraction of the liquid flowing as an annular film along the wall, W_{LF}/W_L, is determined by withdrawing the film through a porous wall to determine its flow rate or by withdrawing samples from the gas stream to determine the droplet flux. The liquid film thickness is measured by a series of conductance probes placed around the pipe perimeter. In this way, a good measurement of the liquid film distribution is obtained, as well as an average film thickness m. Pressure drop is measured using two wall taps on the bottom of the pipe. The manometer lines are kept filled with liquid by bleeding a small liquid flow through the taps.

In this paper we present measurements of the entrainment fraction, $E = (W_L - W_{LF})/W_L$, for air and water flowing in the 2.54 cm (Dallman 1978) and the 5.08 cm (Laurinat 1982) pipelines. The measurements of pressure drop and film height that were obtained in these two pipelines are presented in another paper (Laurinat *et al.* 1984). Our principal goals are to show the effect of pipe diameter and fluid flow rates on entrainment for horizontal air–water flows and to compare entrainment measurements in vertical and horizontal systems.

Measurements of entrainment, pressure drop and film height for upward flow of air and water in a vertical pipeline have been obtained by Willis (1965), Whalley *et al.* (1973), Gill *et al.* (1963), Collier & Hewitt (1961), Cousins *et al.* (1965), Cousins & Hewitt (1968), and Hinkel (1967). Dallman *et al.* (1979) have recently summarized these entrainment results [see the thesis by Dallman (1978) for a more complete discussion], and have shown that they indicate there is a critical film flow rate, W_{LFC}, below which no further atomization occurs. A consequence of this is that the maximum entrainment possible at very high gas velocities is $E_M = (W_L - W_{LFC})/W_L$. Dallman *et al.* (1979) found that

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measurements at different liquid flow rates collapse on a single curve if they are presented as a plot of E/E_M vs $\rho_G^{1/2}\rho_L^{1/2}U_G^3d_t$. The results showed that E/E_M varies linearly with $\rho_G^{1/2}\rho_L^{1/2}U_G^3d_t$ at low gas velocities and approaches an asymptote of one at large gas velocities.

Horizontal flows differ from vertical flows in that the liquid film is not distributed uniformly around the circumference of the pipe and in that gravity plays an important role in controlling the deposition of droplets on the liquid film (McCoy & Hanratty 1977). Consequently, it is not clear to what extent results obtained for vertical upflows can be applied to horizontal flows. Butterworth & Pulling (1973) and Swanson (1966) presented measurements for film height, entrainment, and pressure drop for air and water flowing in horizontal 3.18 cm and 2.54 pipelines. However, as pointed out by Henstock & Hanratty (1976), these do not cover a wide enough range of conditions to come to any definite conclusion regarding the influence of the pipe diameter and the flow rates of the gas and liquid. Other studies of entrainment in a horizontal pipeline include the measurements of Hoogendoorn & Welling (1965) for air and mineral oil flowing in 5.08 cm and 15.24 cm pipelines, and the measurements of droplet flux in the center of the gas core by Wicks & Dukler (1960) and by Magiros & Dukler (1961).

2. INTERPRETATION OF ENTRAINMENT MEASUREMENTS IN VERTICAL FLOWS

Measurements of liquid distribution in annular flow are conveniently interpreted by considering the amount of liquid entrained in the gas flow to result from a dynamic balance between the rate of atomization of the liquid wall layer, R_A, and the rate of deposition of droplets from the gas core, R_D, both given in the units of mass per unit time per unit area. The rate of deposition is usually approximated in terms of the product of the droplet concentration, C_D, and the dimensional deposition constant, k_D:

$$R_D = k_D C_D. \qquad [1]$$

Assuming that the droplets are travelling at the same velocity as the gas,

$$R_D = \frac{k_D \rho_G W_{LE}}{W_G}, \qquad [2]$$

where W_G is the mass flow rate of gas. Under fully developed conditions, $R_A = R_D$, so that

$$E = \frac{R_A W_G}{\rho_G k_D W_L}. \qquad [3]$$

Woodmansee & Hanratty (1969) showed that atomization occurs through the removal of small wavelets riding on top of large flow surges in the liquid film called roll waves or disturbance waves. They suggested that the entrainment occurs because of a Kelvin–Helmholtz instability of these wavelets. Tatterson (1975) used this notion of a Kelvin–Helmoltz instability and the assumption that the wavelets scale with the height of the roll waves, h_p, to develop an equation for R_A. After making a number of simplifying approximations regarding the wave structure and the relation describing pressure variations over small amplitude waves the following relation is obtained for fluids of low viscosity:

$$\frac{R_A}{\rho_G^{1/2}\rho_L^{1/2}v_G^*} \sim \left(\frac{\rho_G v_G^{*2} m}{\sigma}\right). \qquad [4]$$

If [4] is substituted into [3] and k_D is assumed linearly dependent on the friction velocity, v_G^*, it is found that

$$\frac{C_D}{\rho_L^{1/2}\rho_G^{1/2}} = f\left(\frac{\rho_G v_G^{*2} m}{\sigma}\right). \quad [5]$$

This equation has been used by Hutchinson & Whalley (1973) and by Andreussi & Zanelli (1979) to correlate entrainment data for air and water flowing in a vertical pipe.

Dallman (1978), however, expressed concern that [5] does not satisfactorily describe the effect of pipe diameter on the entrainment results obtained at Harwell. He, therefore, developed a completely empirical approach which is more conveniently used, since it related E to controlled variables. He found that available measurements of R_A for air–water flows can be correlated with the following equation, provided the gas velocity, U_G, is not too close to the critical velocity for onset of atomization and provided that the mass flow in the liquid film, W_{LF}, is small enough that the dependence of R_A on W_{LF} is linear:

$$R_A = k_A(W_{LF} - W_{LFC})\, U_G^2\, \rho_G^{1/2}\, \rho_L^{1/2}. \quad [6]$$

Here W_{LFC} is the critical liquid mass flow rate required for the initiation of atomization at the bulk velocity, U_G, P, the wetted perimeter, and k_A, a dimensional atomization rate constant.

The substitution of [6] into [3] gives

$$E\bigg/\left(1 - \frac{W_{LFC}}{W_L}\right) = \frac{d_t\rho_G^{1/2}\rho_L^{1/2}U_G^3(k_A/4k_D)}{1 + d_t\rho_G^{1/2}\rho_L^{1/2}U_G^3(k_A/4k_D)}, \quad [7]$$

with k_A/k_D having the units of sec^2/kg m and $W_L = W_{LF} + W_{LE}$. This equation implies that once the critical velocity for the initiation of atomization is exceeded the entrainment increases very rapidly with increasing gas velocity. In addition, at high velocities a limiting condition is reached, the "fully entrained atomization region", where the flow in the liquid film equals W_{LFC} and the entrainment does not increase with increasing gas velocity,

$$E = 1 - \frac{W_{LFC}}{W_L}.$$

At large liquid flow rates [7] predicts another limiting condition whereby the entrainment does not increase with increasing liquid flow rate,

$$E = \frac{d_t\rho_G^{1/2}\rho_L^{1/2}U_G^3(k_A/4k_D)}{1 + d_t\rho_G^{1/2}\rho_L^{1/2}U_G^3(k_A/4k_D)}.$$

It is of interest to note that [7] predicts that the region of annular flow between that for the initiation of atomization and that for a "fully entrained atomization region" occupies a narrow band of gas velocities.

The measurements of R_A and R_D for air–water flow up a vertical 9.5 mm pipe (Dallman *et al.* 1979) suggest values of $k_A = 3.5 \times 10^{-6}$ s^2/kg, $W_{LFC}/P = 0.046$ kg/m s and $k_D \simeq 0.18$ m/s. Figure 1 shows a comparison of [7] with entrainment measurements of Cousins *et al.* (1965) in a 9.5 mm pipe obtained 575 pipe diameters from the entry. A value of $k_A/4k_D = 4.9 \times 10^{-6}$ s^3/kg m was used in the calculations. The trend of the data is predicted quite well

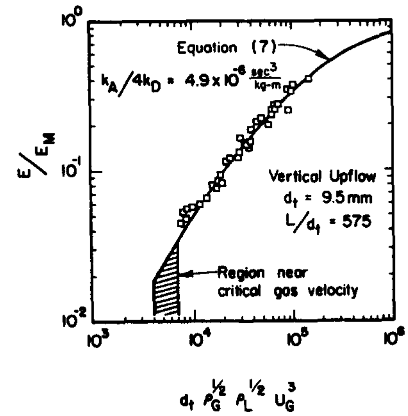

Figure 1. Comparison of entrainment measurements for vertical air–water flows by Cousins *et al.* (1967) with [7].

by theory, indicating that the assumption of constant k_D is a satisfactory approximation for these experiments. Better agreement with data is obtained with $k_A/4k_D = 5.4 \times 10^{-6}$ s^2/kg m. This would indicate that the value of k_A quoted above is 10% too low or the value of k_D is 10% too high.

Measurements of entrainment obtained at 112 and 288 pipe diameters downstream of the entry were also in good agreement with [7] using $k_A/4k_D = 5.4 \times 10^{-6}$/m s and $W_{LFC}/P = 0.046$ kg/m s.

3. DESCRIPTION OF THE EXPERIMENTS

The horizontal system used to carry out these studies was constructed with schedule 80 polyvinyl chloride pipe and the test sections with plexiglas to permit visual observation of the flow. The inner diameters of the plexiglas test sections used in this study were 2.54 and 5.08 cm. They were, respectively, located 550 and 300 pipe diameters from the entry. The PVC pipes had diameters of 2.31 and 4.93 cm. The flanged ends of the pipes were specially designed to insure smooth transitions between sections. Matching tongues and grooves were cut into flange faces so that sections remained concentric at all times. In addition, approximately a 3° taper was cut along the axis on the inside wall of the pipe at the joints. This allowed a close matching of the cross sections of pipes and, therefore, minimized possible flow disturbances at the joints. Water was admitted to the pipeline through an annular slot after the air had flown through a long run of straight pipe.

Film flow rates were measured in the 2.54 cm pipeline by withdrawing the film through the walls of a cylinder of sintered stainless steel with a porosity of 165 μm. The 12.7 cm long cylinder had an inside diameter of 2.54 cm and a wall thickness of 1.6 mm. The porous tube was surrounded by a jacket maintained at a lower pressure than the two-phase flow. The porous pipe was located in the downstream end of a withdrawal unit 150 cm long that was carefully matched to the PVC piping. Measurements were made with this withdrawal section at 230 and at 500 pipe diameters from the entry.

The water film and some air flow through the porous tube wall into the jacket. The mixture in the jacket is channeled to a separator and demister where the water is separated from the mixture and forced by the system pressure through a control valve into a container for measurements. A continuous measurement of the air withdrawal rate was accomplished by passing the air vented from the separator through a rotameter.

Previous studies by Jagota *et al.* (1973) and by Anderson & Russell (1970) in a vertical system indicated that a length of porous wall of 7.6 cm was sufficient to remove all of the liquid film. However, in a horizontal system the length of porous wall needed is much less at the top of the tube than at the bottom because of the nonuniform film thickness distribution around the pipe circumference. This problem was overcome in the 2.54 cm pipe by drilling an array of 0.45 mm holes in the lower 150° of tube wall to increase the porosity on the bottom half relative to that in the top half.

Measurements of entrainment in the 5.08 cm pipeline were made 300 pipe diameters from the entry. A 14 cm sintered bronze cylinder with a porosity of 170 μm was used. For thin liquid films it was found that a porous length of 2.5 cm was all that was needed. In these cases the downstream end of the withdrawal unit was wound with a tape.

For thicker, more asymmetric films, another 170 μm cylinder with 3.2 mm holes drilled in the bottom 72° was used. The spacing of these holes was much greater than what was used in the 2.54 cm withdrawal unit. They were located at a 5.08 mm pitch along the length of the cylinder and were placed 12° apart in a staggered formation. These added holes accounted for about 23% of the surface area of the portion of the cylinder in which they were located. All but the first 2.54 cm of the top 260° of the cylinder was covered with strips of teflon tape to prevent excessive air flow through the cylinder.

Detailed studies of the effect of air withdrawal rate on the measured film flow rate were performed. Staniforth *et al.* (1965) showed for vertical air–liquid flows that the liquid withdrawn through a porous wall varies linearly with the air withdrawn until the wall layer has been removed, and then the liquid rate increases at a much slower rate with increasing air withdrawal. This was also the case for the results of this study. In fact, tests such as this were used as a criterion for a properly designed withdrawal unit. For the 2.54 cm pipe it was found that about 2% of the air had to be withdrawn to remove the entire liquid film.

An additional check on the film withdrawal techniques was obtained by measuring droplet fluxes over the pipe cross section. The pitot tubes used to measure droplet fluxes were held by a traversing mechanism which allowed the positioning of the pitot tube along diameters at 0°, 45° and 90°. Suction was applied to the end of the pitot tube and a mixture of air and water was withdrawn from the flowing stream. The flux of liquid was determined by dividing the rate of liquid withdrawal by an area based on the outside diameter of the pitot tube. Tatterson (1975) found that the suction flow rate has almost no effect on the measured droplet flux, provided the suction velocity is greater than 10% of the isokinetic velocity. Consequently, it was not necessary to control the air withdrawal rate carefully.

The pitot tube used in the 2.54 cm pipe was made from 3 mm o.d. by 1.8 mm i.d. stainless steel tubing. It extended 10 cm upstream to avoid any disturbances caused by the entry of the tube through the side of the pipe. The outer wall of the pitot tube was sanded to a knife edge at the end facing the flow. Two pitot tubes were used in the 5.08 cm pipeline. One had an inside diameter of 1.75 cm and the other an inside diameter of 3.86 mm. The droplet fluxes measured with these two tubes were found to be the same.

The total mass flow of entrained liquid in the gas, W_{LE}, was obtained from mass flux profile measurements made using six rotations of the pitot tube traversing mechanism. For the first profile, taken at 0° to the vertical, the centerline could be traversed from 1.5 mm from the bottom wall to 6 mm from the top wall. By rotating the mechanism 180° fluxes closer to the top wall could be obtained. The procedure was repeated at 45° and 90° to the vertical.

4. RESULTS

(a) *Mass flux profiles*

Figures 2 and 3 show liquid mass fluxes in the gas phase for two widely different flow conditions. In figure 2 the small circle indicates how the measurements were obtained. Zero and one are the pipe walls. The numbers along the abscissa are the fraction of the diameter from the zero wall.

The profiles exhibit an approximate bilateral symmetry. The one at 90° shows a

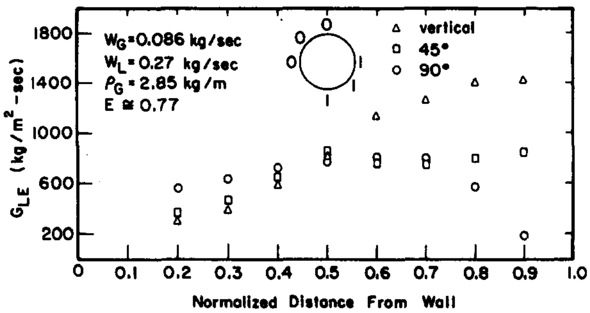

Figure 2. Measurements of local droplet flux in a horizontal 2.54 cm pipe; W_G = 0.086 kg/s, W_L = 0.049 kg/s.

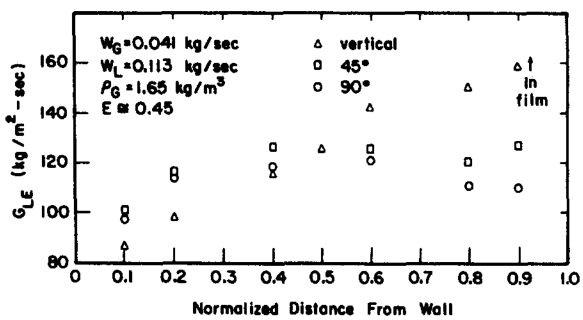

Figure 3. Measurements of local droplet flux in a horizontal 2.54 cm pipe; W_G = 0.041 kg/s, W_L = 0.113 kg/s.

decreased liquid mass flux near the liquid film with a maximum at the pipe center. The 45° and vertical profiles show parallel increase in the liquid mass flux as the pitot tube traverses the pipe. However increases in the mass flux for the 45° profiles tend to become small after the pipe center is reached and before contact with the liquid layer is made. The vertical profile continues to increase at about the same rate across the pipe diameter until the liquid film is intercepted.

It is of interest to note that for the flow conditions examined nearly twice as much entrained mass is flowing in the bottom half of the pipe as in the top half. Possible explanations for this are the larger atomization rate for the thick films on the bottom of the pipe and the larger deposition rates in the bottom half because of the influence of gravity on the droplet motion.

The integration of these local droplet fluxes gives a total flow rate of droplets in the gas phase. A comparison of measurements of entrainment obtained in this manner with measurements obtained by film withdrawal is given in table 1. The good agreement provides support that measurements using film withdrawal are accurate. Almost all of the entrainment measurements reported in the results section were determined using film withdrawal. The pitot tube was used only in a few runs in the 5.08 cm pipe at low gas velocities near the transition form annular to stratified flow.

(b) *Effect of fluid velocities, and gas density*

The effect of gas velocity on entrainment is shown in figure 4 where E is plotted vs $\rho_G^{1/6}U_G$ for the 2.54 cm horizontal pipeline, as suggested by [7]. It is noted that at large gas velocities the entrainment reaches a constant value, which increases with increasing liquid throughput. This indicates that for a fixed liquid rate the flow reaches a fully

Table 1. Comparison of entrainments in the 2.54 cm pipe measured with pitot tubes and with film withdrawal units

W_G kg/sec	W_L kg/sec	ρ_G kg/m^3	E_P Pitot tube	E_{FW} Film withdrawal
0.086	0.049	2.85	0.77	0.75
0.041	0.113	1.65	0.45	0.47
0.086	0.0210	2.85	0.64	0.61

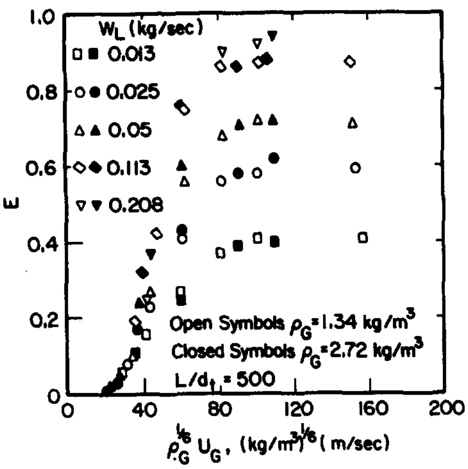

Figure 4. Effect of air velocity on entrianment for air and water flowing in a horizontal 2.54 cm pipeline at 500 pipe diameters from the entry.

entrained condition where it is not possible to decrease the film flow rate with further increase in gas velocity. A twofold change in the gas density is found to have little effect on entrainment provided comparisons are made at the same gas velocity.

The effect of increasing liquid flow rate on entrainment is shown in figure 5, where E is plotted against W_L for the 2.54 cm horizontal pipeline. It is noted that for a fixed gas velocity the entrainment increases with increasing liquid flow rate and reaches a plateau at high liquid flow rate, as predicted by [7].

(c) *Critical film flow rates*

The measurements shown in figure 4 suggest that there is a critical film flow rate below which atomization does not occur. This can be calculated by comparing maximum values of the entrainment, such as are shown in figure 4, with the equation $E = (W_L - W_{LFC})/W_L$.

A consideration of values of W_{LFC} obtained in this manner indicates that, unlike what is found for vertical flow, they are a function of the flow rate of entrained liquid. This is illustrated in figure 6 where measured values of the film flow rate are plotted against W_{LE}/W_G for the 5.08 cm pipe. The curve in the figure represents an approximate lower limit for W_{LF}.

This curve is compared with a similar one obtained from measurements in the 2.54 cm pipe in figure 7. The film flow rate observed by Woodmansee & Hanratty (1969) for the

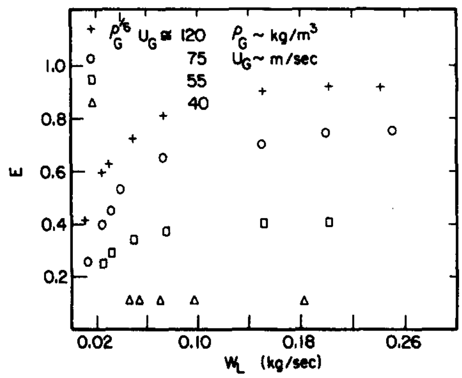

Fig. 5.

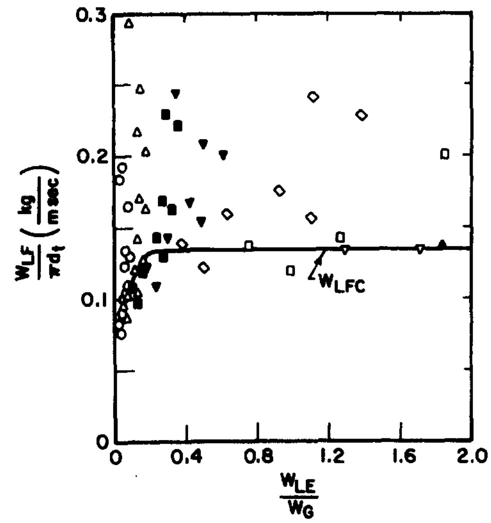

Fig. 6.

Figure 5. Effect of liquid flow rate on entrainment for air and water flowing in a horizontal 2.54 cm pipeline at 500 pipe diameters from the entry.

Figure 6. Evaluation of W_{LFC} from a plot of W_{LF} vs W_{LE}/W_G.

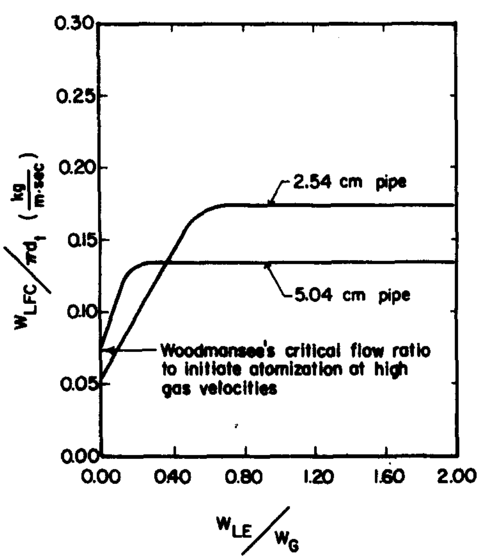

Figure 7. Critical film flow rates for air and water flowing in horizontal 2.54 and 5.08 cm pipelines.

onset of atomization is also indicated in this figure. It is to be noted that this is in good agreement with measurements of W_{LFC}/P for $W_{LE}/W_G \to 0$.

(d) *Entrainment correlation*

A summary of the film flow rate measurements obtained in the 2.54 cm and the 5.08 cm pipelines is given in figures 8 and 9. These results shown a rapid decrease in the film flow rate with increasing gas velocity until at large enough velocities the film flow rate reaches W_{LFC}. We call this flow configuration at high gas velocities the "fully entrained atomization region".

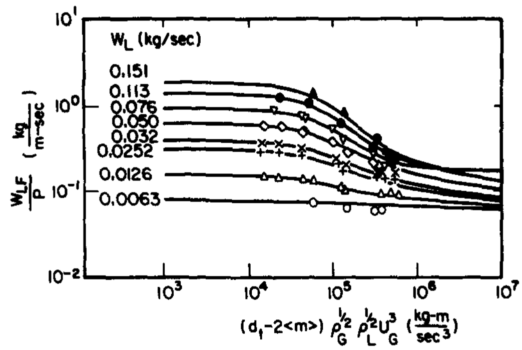

Figure 8. Measurements of the film flow rate for air and water flowing in a 2.54 cm horizontal pipeline.

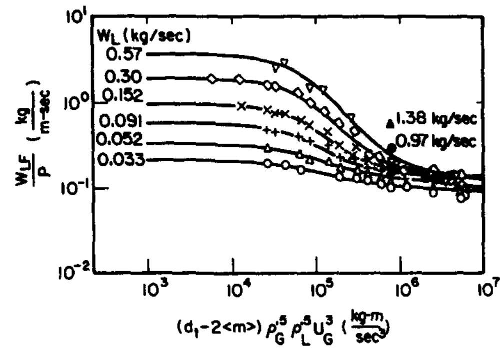

Figure 9. Measurements of the film flow rate for air and water flowing in a 5.08 cm horizontal pipeline.

Values of the entrainment are calculated from the results such as those shown in figures 8 and 9 since $E = (W_L - W_{LF})/W_L$. Results obtained at 500 diameters and 230 diameters from the entry are plotted in figures 10 and 11 for the 2.54 cm pipe, using the values of W_{LFC} given in figure 7. The plots are in agreement with [7] in that no effect of liquid throughput is seen if E/E_M is plotted againt $\rho_G^{1/2}\rho_L^{1/2}U_G^3d_t$. The same results are obtained at 230 diameters as at 500 diameters, indicating that a fully developed condition is reached at 230 diameters such that changes in the entrainment with distance downstream occur mainly through changes in the gas velocity associated with the decrease in gas pressure.

The results in figures 10 and 11 show that the entrainment increases with U_G^3 until a fully entrained condition is reached. Thus, once the critical gas velocity for atomizing the liquid is exceeded, the entrainment increases very rapidly with increasing gas velocity, and the region between the initiation of atomization and the fully entrained condition covers only a threefold change in gas velocity. This observed increase of E with increasing gas velocity is much more rapid than would be predicted by [7] with $k_A/4k_D$ a constant. This is illustrated in figure 10, where [7] is plotted using $k_A/4k_D = 3.0 \times 10^{-6}\ s^2/kg\ m$.

An empirical fit to the data shown in figures 10 and 11 is given by

$$\frac{E}{1-\dfrac{W_{LFC}}{W_L}} = \frac{3.6\times10^{-8}[(d_t-2\langle m\rangle)\rho_G^{1/2}\rho_L^{1/2}U_G^3]^{1.5}}{1+3.6\times10^{-8}[(d_t-2\langle m\rangle)\rho_G^{1/2}\rho_L^{1/2}U_G^3]^{1.5}}, \qquad [8]$$

where W_{LFC} is given by figure 7 and 3.6×10^{-8} is a dimensional constant with units of $s^{4.5}/m^{1.5}\ kg^{1.5}$.

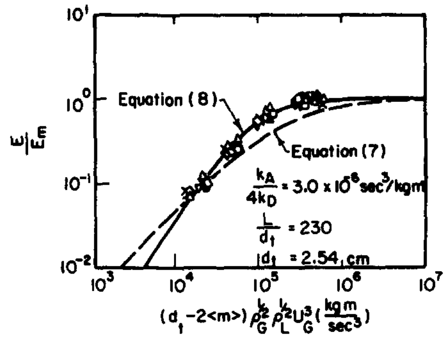

Fig. 10.

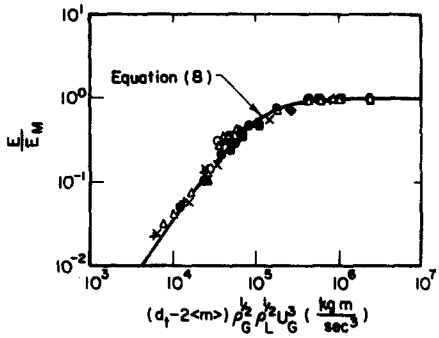

Fig. 11.

Figure 10. Correlation of entrainment measurements for air and water flowing in a horizontal 2.54 cm pipeline, taken at $L/d_t = 230$.

Figure 11. Correlation of entrainment measurements for air and water flowing in a horizontal 2.54 cm pipeline, taken at $L/d_t = 500$.

Entrainment results obtained in the 5.08 cm pipe at $300L/d_t$ are summarized in figure 12. A comparison of these results with those shown in figures 10 and 11 indicates that little effect of pipe diameter is observed if the results are plotted in the manner suggested by [7].

5. INTERPRETATION

(a) *Effect of flow variables and pipe diameter*

For the limited range covered in this research, the effect of gas density is found to be small provided comparisons of measured values of entrainment are made at the same gas velocity. The effects of a change of gas velocity are much stronger.

Once the gas velocity exceeds the critical gas velocity the entrainment increases with the cube of the gas velocity until at large enough gas velocities the fully entrained atomization region is reached where entrainment levels to a constant value less than unity. The reason why not all the liquid is entrained is that there is a critical film flow rate below which no further atomziation occurs. This critical film flow rate is found to vary weakly with the concentration of liquid in the gas, W_{LE}/W_G, and to be approximately independent of pipe diameter, if expressed as a mass flow per unit perimeter, $W_{LFC}/\pi d_t$.

The rate equations used to interpret the results assume a linear dependence of the rate of atomization on $W_{LF} - W_{LFC}$ and a linear dependence of the rate of deposition on W_{LE}/W_G. The good agreement of the measurements of E/E_M at a fixed gas velocity for a wide range

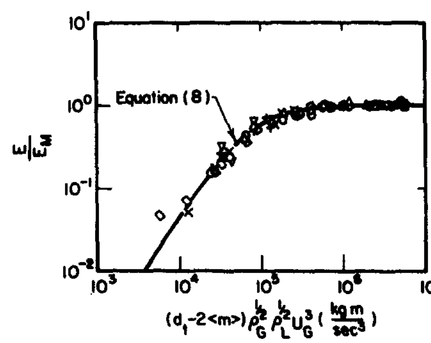

Figure 12. Comparison of entriainment measurements for the 5.08 cm pipe at $300L/d_t$ with [8]. Key is the same as for figure 9.

of liquid inputs implies that [2] and [6] correctly represent the effect of liquid flow rate. These equations predict that the entrainment is independent of liquid flow if the critical film flow rate is zero. This is because of the linear dependencies of R_A and R_D on W_{LF} and W_{LE}. An increase in W_L causes proportionate changes in W_{LF} and W_{LE} but no change in $E = W_{LE}/(W_{LF} + W_{LE})$. Because the critical film flow rate is not zero, there is an effect of W_L on W_{LE} at low liquid throughputs. This can be represented by assuming $W_{LE}/(W_L - W_{LFC})$ is constant at constant gas velocity.

The good agreement of the measurements for the 2.54 cm and the 5.08 cm pipelines plotted in figures 10–12 indicates that entrainment measurements for different pipe sizes will be independent of pipe diameter if plotted as E/E_M vs $(d_t - 2\langle m\rangle)\rho_G^{1/2}\rho_L^{1/2}U_G^3$. It thus appears that for horizontal pipelines the ratio of the rate constants, k_A/k_D, is a weak function of pipe diameter.

(b) *Comparison with measurements in a vertical pipe*

It has been found that the same type of plot can represent the effect of liquid and gas flow rates on entrainment in vertical and horizontal pipes. However, there are marked differences in the entrainment measurements for the two systems.

As is shown in figure 1, measurements in a vertical pipe for a given fluid pair can be represented reasonably well by [7] with k_A/k_D constant. A comparison of measurements in a horizontal pipe with [7] for $4k_A/k_D = 3.0 \times 10^{-6}\ \mathrm{s^2/kg}$ is given in figure 10. It is noted that the observed increase of entrainment with increasing gas velocity is much more rapid than what is predicted by [7]. A comparison of [7] with the empirical correlation, [8], indicates that

$$k_A/k_D \sim d_t^{0.5}u_G^{1.5}. \qquad [9]$$

The increase of k_A/k_D with increasing gas velocity in horizontal pipes can be explained if k_D is strongly affected by gravitational settling. An additional characteristic velocity, the terminal settling velocity of droplets, then enters the description of the problem. In fact, under certain circumstances it can be argued that k_D/V_t is a constant (McCoy & Hanratty 1977). Assume that this is the case for the measurements presented in this paper. An increase in gas velocity would be accompanied by a decrease in droplet size and, therefore, a decrease in V_T. If k_D/V_T is constant, one would then expect a decrease in k_D and an increase in k_A/k_D with increasing gas velocity.

Another difference in entrainment measurements in vertical and horizontal pipes is the effect of pipe diameter. The measurements presented in this paper indicate k_A/k_D varies as $d_t^{0.5}$. Recent studies by Asali (1983) with 1.905 and 3.81 cm pipes and the summary of literature data by Dallman *et al.* (1979) indicate a much stronger effect of pipe diameter on k_A/k_D in vertical systems. This indicates that different mechanisms for deposition could be operative in vertical and horizontal systems.

Finally, in discussing differences in entrainment measurements in vertical and horizontal systems, it should be mentioned that the effect of W_{LE}/W_G on W_{LFC}, found only in horizontal pipes, is not well understood. It is probably associated with the asymmetry of the distributions of the liquid film and the droplets.

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NOTATION

C_D concentration of droplets in the units of mass per unit volume

d_t tube diameter

E fraction of liquid entrained $= W_{LE}/W_L$
E_M maximum possible entrainment $= (W_L - W_{LFC})/W_L$
k_D deposition rate constant defined by [2]
k_A atomization rate constant defined by [6]
m average film height around the pipe circumference
p pipe perimeter $= \pi d_t$
R_A rate of atomization in units of mass per unit time per unit area
R_D rate of deposition in units of mass per unit time per unit area
U_G gas velocity
v_G^* friction velocity for the gas
V_T terminal velocity of a settling drop
W_G mass flow rate of the gas
W_L mass flow rate of the liquid
W_{LE} mass flow rate of the entrained liquid
W_{LF} mass flow rate of the liquid film
W_{LFC} critical film flow rate
ρ_G gas density
ρ_L liquid density
σ surface tension
$\sim$ is a function of

REFERENCES

ANDERSON, R. J. & RUSSELL, T. W. F. 1970 Film formation in two-phase annular flow. *AIChE J.* **16**, 626–632.

ANDREUSSI, P. & ZANELLI, S. 1979 Downward annular and annular-mist flow of air-water mixtures. *Two-Phase Momentum, Heat and Mass Transfer in Chemcial, Process and Energy Engineering Systems* (Edited by F. DURST, G. V. TSIKLAURI and N. AFGAN), Vol. 1, p. 303. Hemisphere, Washington, D.C.

ASALI, J. C., 1983 Ph.D. Thesis, University of Illinois, Urbana.

BUTTERWORTH, D. & PULLING, D. J. 1973. Film flow and film thickness measurements for horizontal annular air–water flow. *AERE-R*7576.

COLLIER, J. C. & HEWITT, G. F. 1961 Data on the vertical flow of air–water mixtures in the annular and dispersed flow regions. *Trans. Inst. Chem. Engrs* **39**, 127–136.

COUSINS, L. B., DENTON, W. H. & HEWITT, G. F. 1965 Liquid mass transfer in annular two-phase flow. Paper presented at the Symp. on Two-Phase Flow, Exeter, England, 21–23 June.

COUSINS, L. B. & HEWITT, G. F. 1968 Liquid phase mass transfer in annular two-phase flow: Droplet deposition and liquid entrainment. *AERE-R*75657.

DALLMAN, J. C., JONES, B. G. & HANRATTY, T. J. 1979 Interpretation of entrainment measurements in annular gas–liquid flows. *Two-Phase Momentum, Heat and Mass Transfer in Chemical Process and Energy Engineering Systems* (Edited by F. DURST, G. V. TSIKLAURI and N. AFGAN), Vol. 2, p. 681. Hemisphere, Washington, D.C.

DALLMAN, J. C. 1978 Ph.D. Thesis, Investigations of separated flow model in annular gas–liquid two-phase flows. University of Illinois, Urbana.

GILL, L. E., HEWITT, G. F. & LACEY, P. M. C. 1963 Sampling probe studies of the gas core in annular flows. *Chem. Engng Sci.* **19**, 665–680.

HENSTOCK, W. H. & HANRATTY, T. J. 1976 The interfacial drag and the height of the wall layer in annular flows. *AIChE J.* **22**, 990–1000.

HINKEL, .W D. 1967 A study of liquid mass transport in annular air–water flow. Ph.D. Thesis, Massachusetts Institute of Technology, Cambridge.

HOOGENDOORN, C. J. & WELLING, W. W. 1965 Experimental studies on the characteristics of the annular-mist flow in horizontal pipes. Paper presented at the Symp. on Two-Phase Flow, Exeter, England, 21–23 June.

HUTCHINSON, P. & WHALLEY, P. B. 1973 A possible characterization of entrainment in annular flows. *Chem. Engng Sci.* **28**, 974–975.

JAGOTA, A. K., RHODES, E. & SCOTT, D. S. 1973 Tracer measurements in two-phase annular flow to obtained interchange and entrainment. *Can. J. Chem. Engrs* **51**, 139–148.

LAURINAT, J. E. 1982 Studies of the effect of pipe size on horizontal annular two-phase flow. Ph.D. Thesis in Chemcial Engineering, University of Illinois, Urbana.

LAURINAT, J. E., HANRATTY, T. J. & DALLMAN, J. C. 1984 *Int. J. Multiphase Flow* **10**, 341–356.

MAGIROS, P. G. & DUKLER, A. E. 1961 Entrainment and pressure drop in concurrent gas–liquid flow—II. Liquid property and momentum effects. *Developments in Mechanics*, Vol. 1. Plenum Press, New York.

MCCOY, D. D. & HANRATTY, T. J. 1977 Rate of deposition of droplets in annular two-phase flow. *Int. J. Multiphase Flow* **3**, 319–331.

SWANSON, R. W. 1966 Characteristics of the gas–liquid interface in two-phase annular flow. Ph.D. Thesis, University of Delaware, Wilmington.

STANIFORTH, R., STEVENS, G. F. & WOOD, R. W. 1965 An experimental investigation into the relationship between burnout and film flow rate in a uniformly heated round rod. *AEEW-R*-430.

TATTERSON, D. F. 1975 Rates of atomization and drop size in annular two-phase flow. Ph.D. Thesis, University of Illinois, Urbana.

TATTERSON, D. F., DALLMAN, J. C. & HANRATTY, T. J. 1977 Drop sizes in annular gas–liquid flows. *AIChE J.* **23**, 68–76.

WHALLEY, P. B., HEWITT, G. F. & HUTCHINSON, P. 1973 Experimental wave and entrainment measurements in vertical annular two-phase flow. *AERE-R*7521.

WICKS, M. & DUKLER, A. E. 1960 Entrainment and pressure drop in concurrent gas–liquid flow. *AIChE J.* **6**, 463–468.

WILLIS, J. J. 1965 Upwards annular two-phase air–water flow in vertical tubes. *Chem. Engng Sci.* **20**, 895–902.

WOODMANSEE, D. E. & HANRATTY, T. J. 1969 Mechanism for the removal of droplets from a liquid surface by a parallel air flow. *Chem. Engng Sci.* **24**, 299–307.